

Lattice Hamiltonian approach to the Schwinger model

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Outline

1. The Schwinger model on lattice
2. Strong coupling expansion (SCE)
3. Ground state energy
4. Mass gaps
5. Chiral condensate
6. Oscillations of chiral condensate
7. Summary & outlook

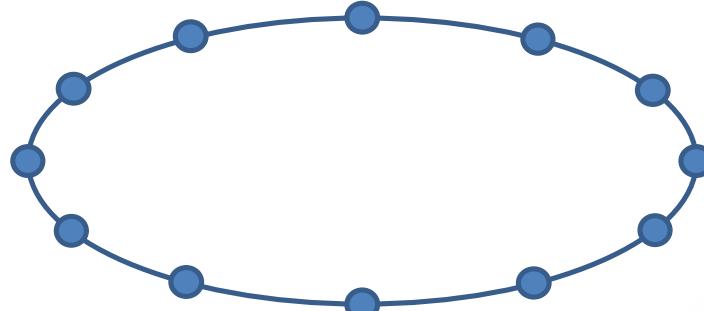


The Schwinger model

Hamiltonian of the Schwinger model in the Kogut-Susskind staggered discretization [1,2]:

$$\mathcal{H} = -\frac{i}{2a} \sum_{n=1}^M \left(\phi^\dagger(n) e^{i\theta(n)} \phi(n+1) - \phi^\dagger(n+1) e^{-i\theta(n)} \phi(n) \right)$$
$$+ m \sum_{n=1}^M (-1)^n \phi^\dagger(n) \phi(n) + \frac{ag^2}{2} \sum_{n=1}^M L^2(n)$$

- $\phi(n)$ – single-component fermion field on a circle with M sites
- $\theta(n) = agA_1(n)$ – gauge field variable related to the Abelian vector potential
- $L(n) = E(n)/g$ – variable related directly to the electric field
- m – fermion mass
- a – lattice spacing
- g – gauge coupling constant



[1] Kogut and Susskind, Phys. Rev. D 11 (1975) 395

[2] Banks, Susskind and Kogut, Phys. Rev. D 13 (1976) 1043

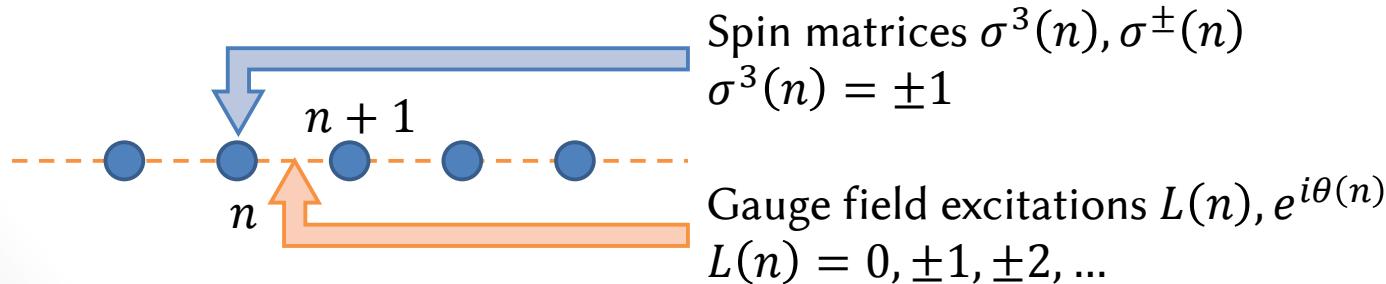


The Schwinger model

Hamiltonian of the Schwinger model in lattice representation after the Jordan-Wigner transformation [3]:

$$\begin{aligned}\mathcal{H}_{JW} = & -\frac{1}{2a} \sum_{n=1}^M (\sigma^+(n) e^{i\theta(n)} \sigma^-(n+1) + \text{h.c.}) \\ & + \frac{m}{2} \sum_{n=1}^M (1 + (-1)^n \sigma^3(n)) + \frac{ag^2}{2} \sum_{n=1}^M L^2(n)\end{aligned}$$

- $\sigma^i(n)$ – Pauli matrices residing on the sites
- $L(n)$ – gauge field excitations defined between sites n and $n+1$
- $e^{\pm i\theta(n)}$ – ladder operators for gauge field excitations



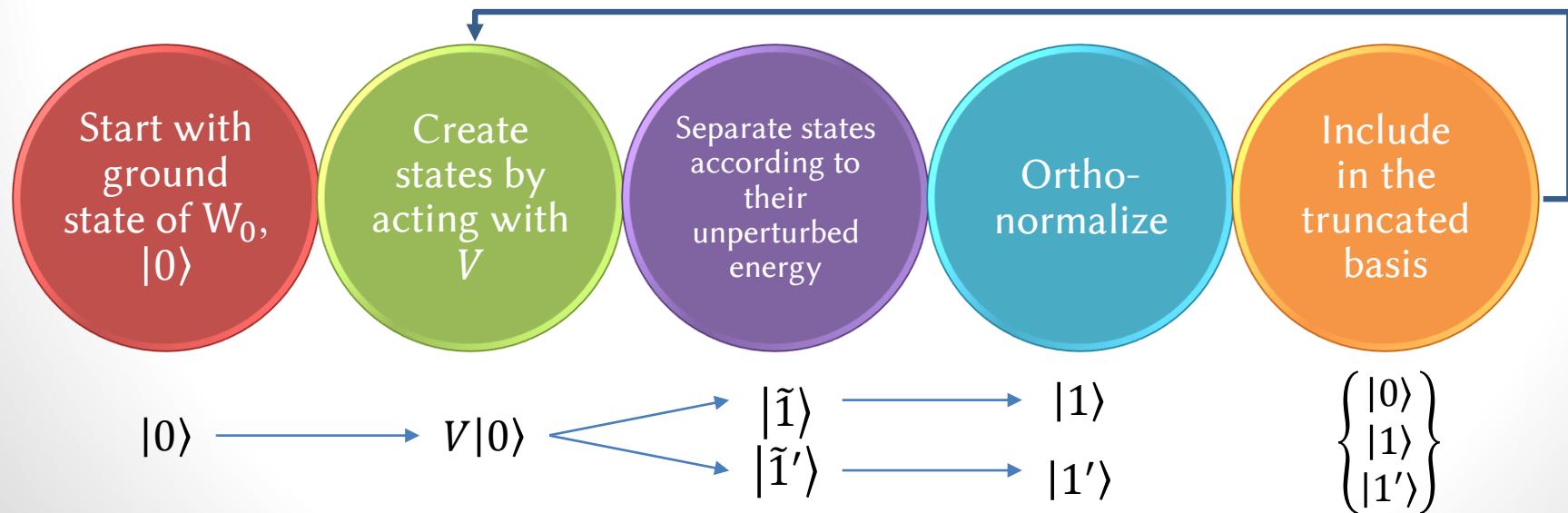


Strong coupling expansion on the Schwinger model

Rewrite the Hamiltonian in a dimensionless form:

$$W = \frac{2}{ag^2} \mathcal{H}_{JW} = W_0 + xV \quad \begin{array}{l} \xrightarrow{\quad} \sum_{n=1}^M (\sigma_n^+ e^{i\theta(n)} \sigma_{n+1}^- + \text{h.c.}) \\ \xrightarrow{\quad} \frac{m}{ag^2} \sum_{n=1}^M (1 + (-1)^n \sigma_n^3) + \sum_{n=1}^M L^2(n) \end{array}$$

- If $x \equiv \beta = \frac{1}{a^2 g^2}$ is small, we can treat W_0 as an unperturbed Hamiltonian and V as a perturbation.
- SCE creates the truncated basis of W





Observables

- Ground state energy:

$$E_0 = \frac{\omega_0}{2Mx} \xrightarrow[M \rightarrow \infty]{a \rightarrow 0} -\frac{1}{\pi}$$

- Scalar mass gap ($m = 0$):

$$\frac{M_S}{g} = \frac{\omega_1 - \omega_0}{2\sqrt{x}} \xrightarrow[M \rightarrow \infty]{a \rightarrow 0} \frac{2}{\sqrt{\pi}}$$

- Vector mass gap ($m = 0$):

$$\frac{M_V}{g} = \frac{\omega_0^V - \omega_0}{2\sqrt{x}} \xrightarrow[M \rightarrow \infty]{a \rightarrow 0} \frac{1}{\sqrt{\pi}}$$

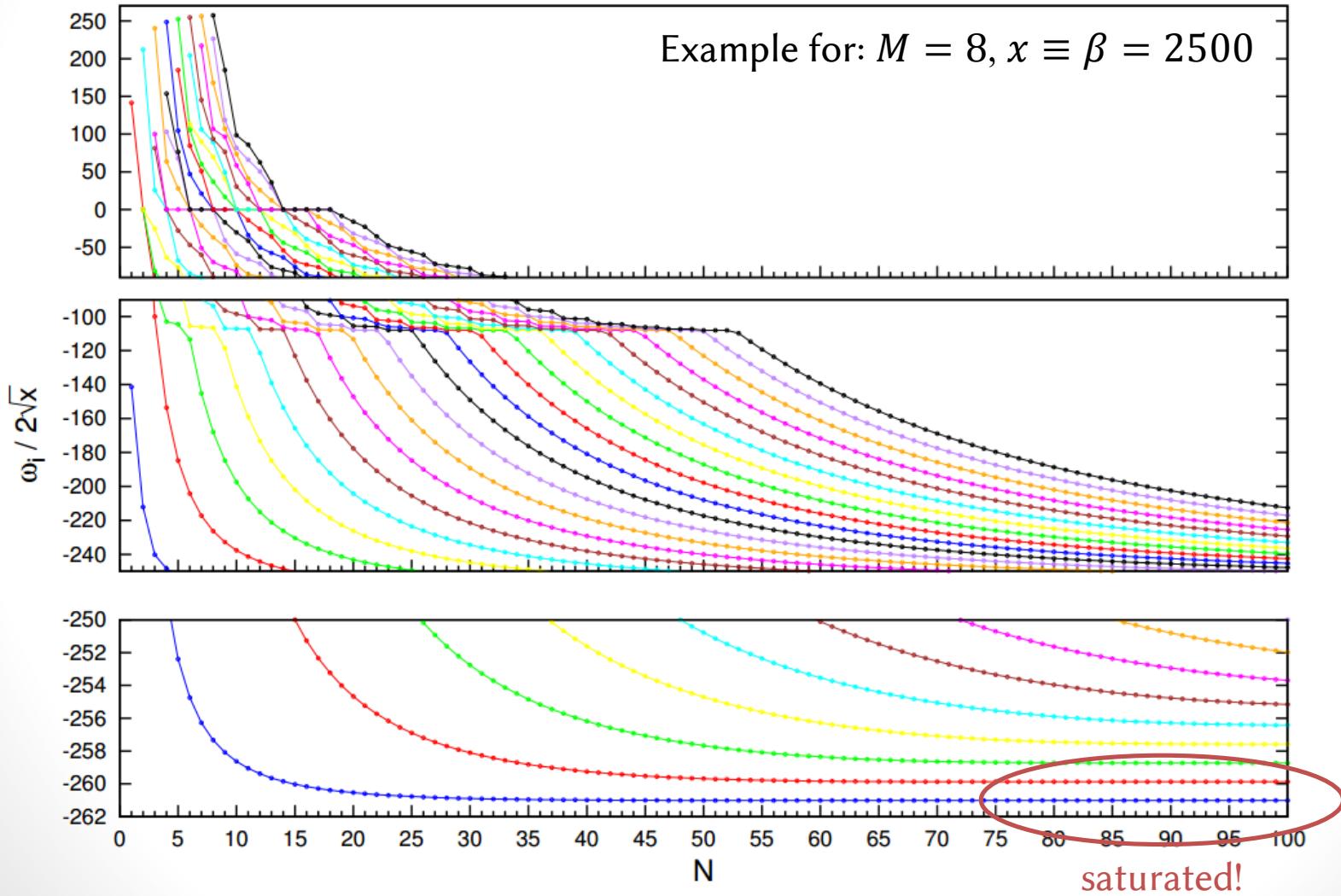
- Chiral condensate (chiral order parameter):

$$\frac{\langle \bar{\psi}\psi \rangle_0}{g} = \frac{\sqrt{x}}{2M} \langle 0 | \sum_{n=1}^M (-1)^n \sigma^3(n) | 0 \rangle$$

- ω_i – eigenvalues of W_0
- ω_i^V – eigenvalues of vector Hamiltonian created using SCE with first state $V^-|0\rangle$.



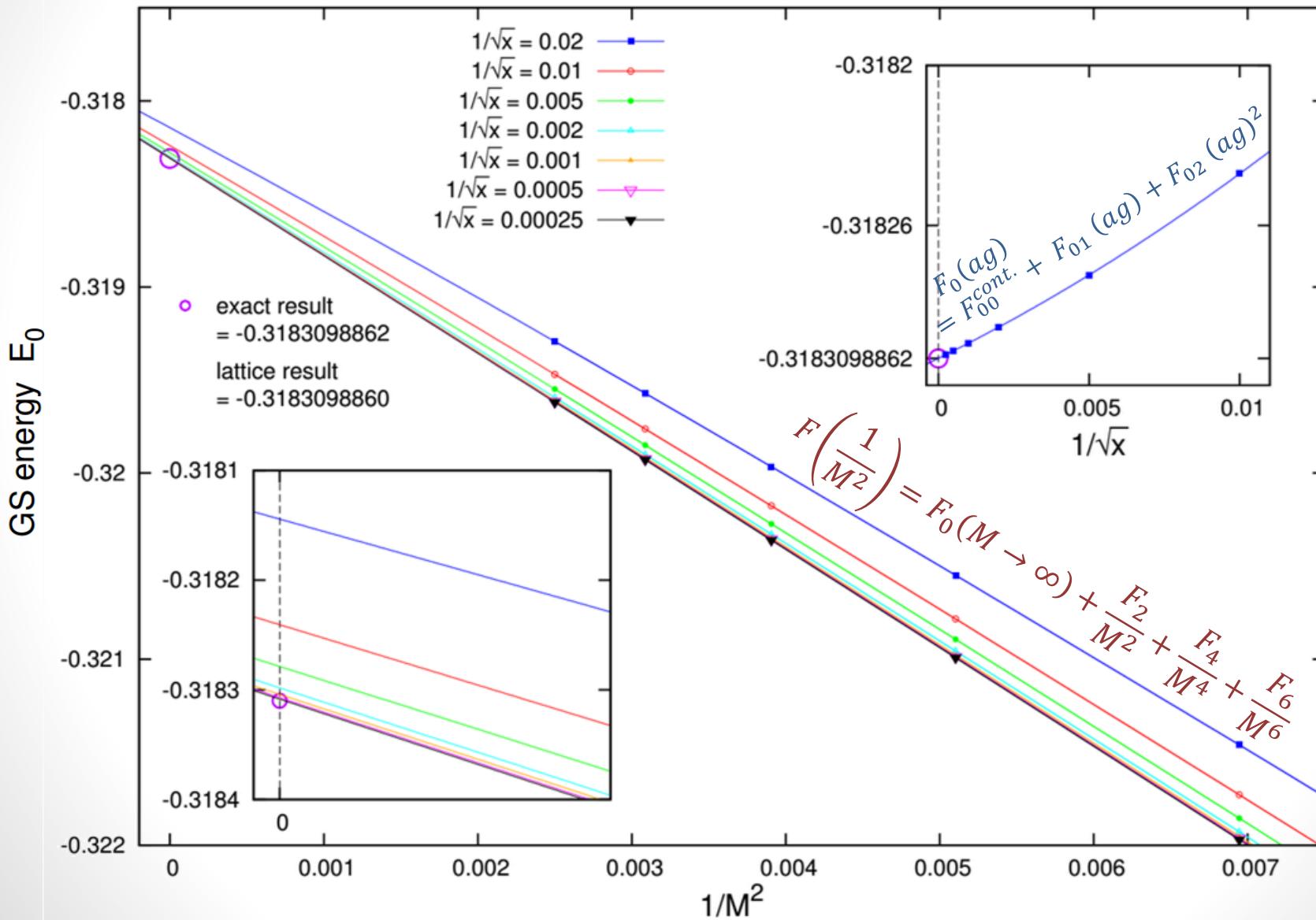
Eigenvalue flow with the order of strong coupling expansion, N



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Ground state energy

$$E_0 = \frac{\omega_0}{2Mx} \quad m = 0$$

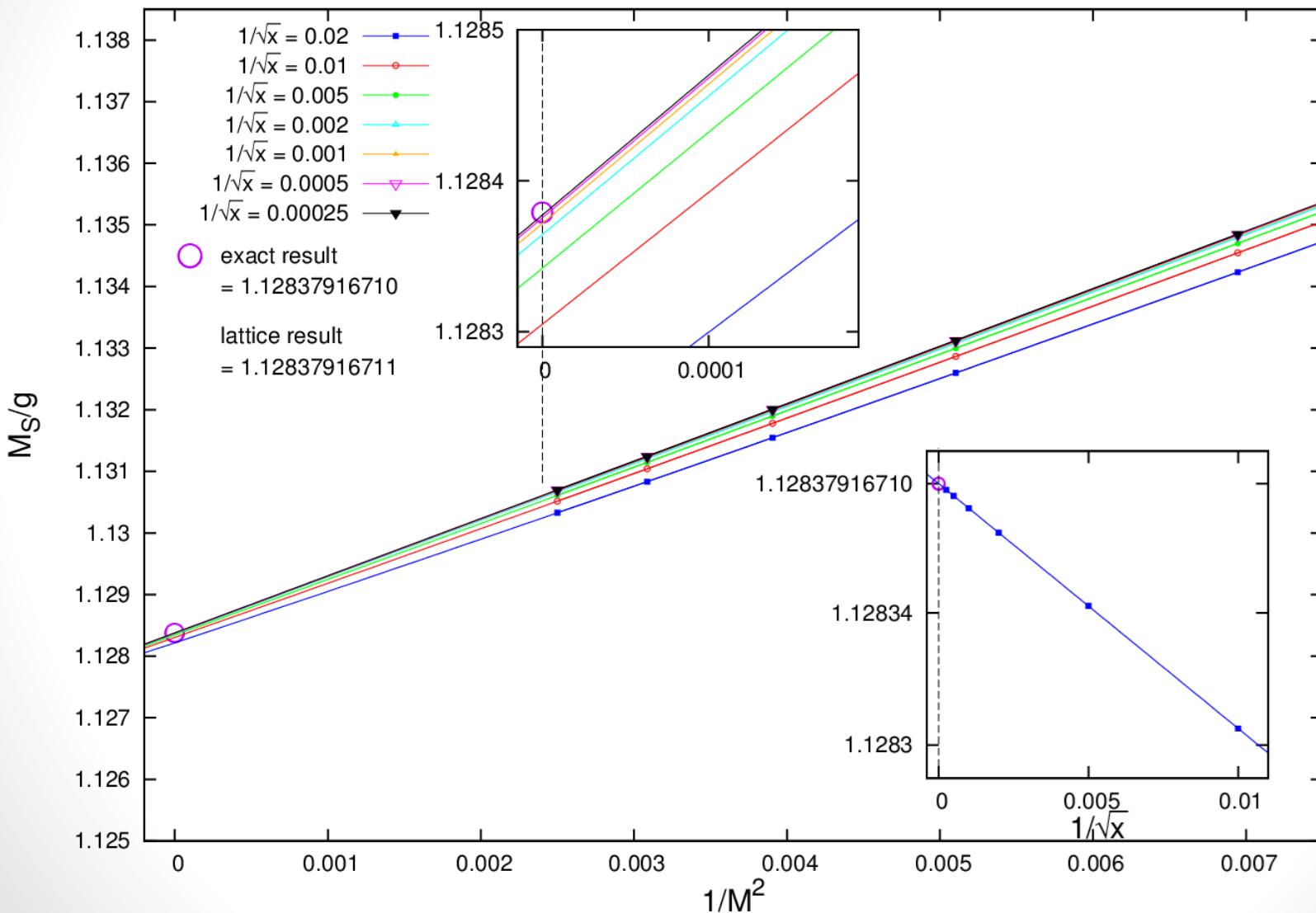




Scalar mass gap

$$\frac{M_S}{g} = \frac{\omega_1 - \omega_0}{2\sqrt{x}}$$

$m = 0$



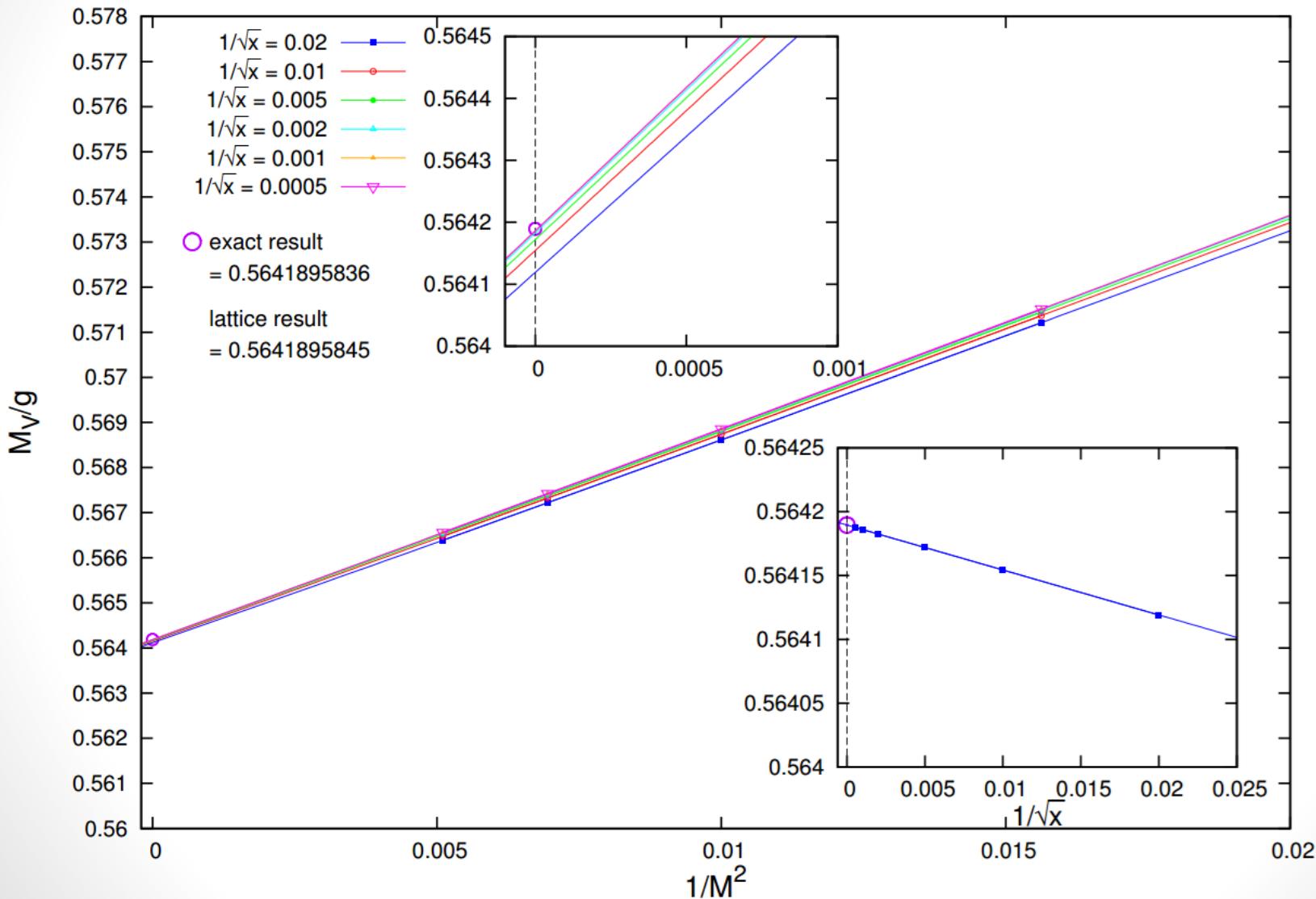
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Vector mass gap

$$\frac{M_V}{g} = \frac{\omega_0^V - \omega_0}{2\sqrt{x}}$$

$m = 0$



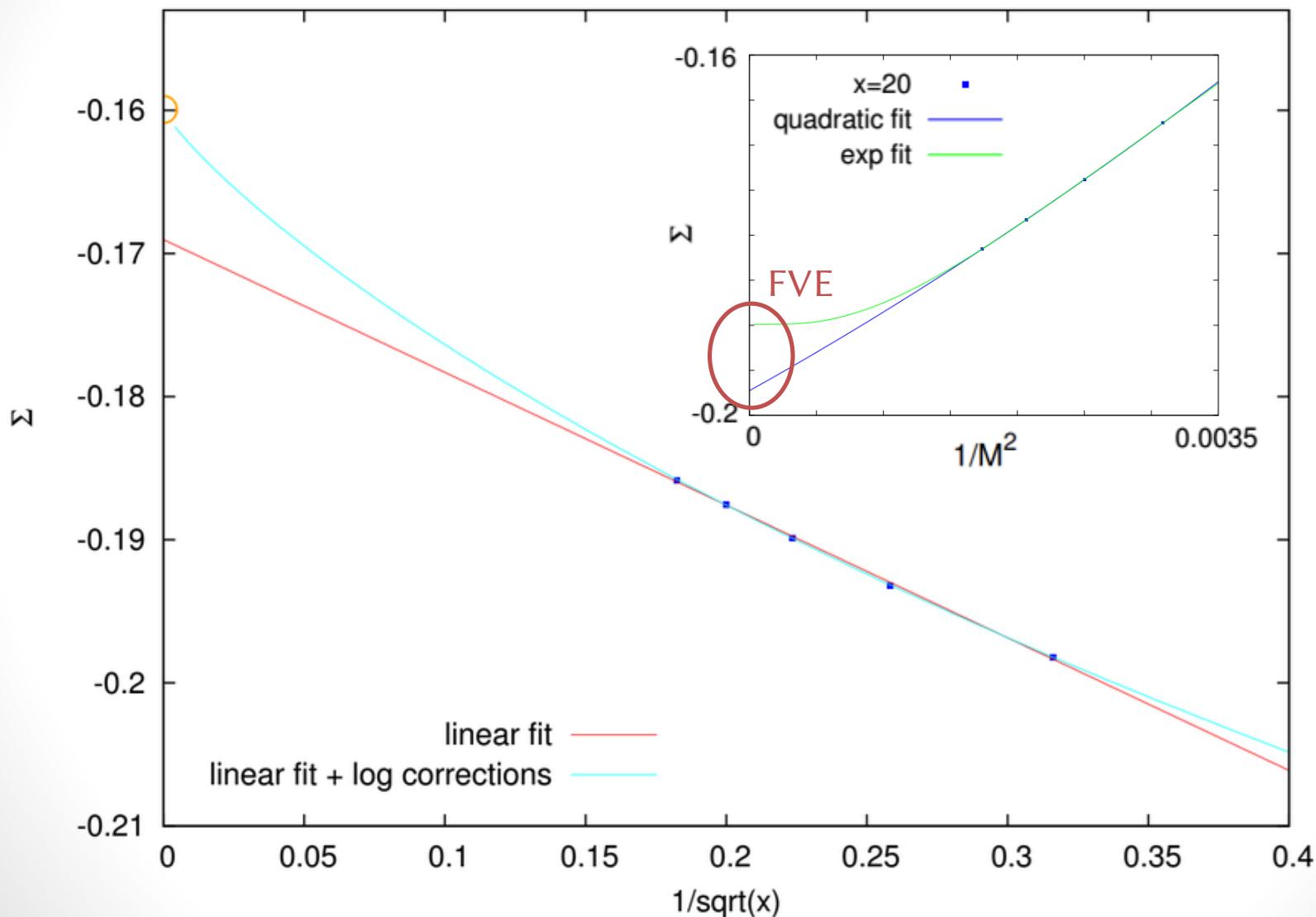
Lattice Hamiltonian approach
to the Schwinger model

(10)



Chiral condensate

$m = 0$



Lattice Hamiltonian approach
to the Schwinger model

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Comparison with MPS results

- Ground state energy and mass gaps – massless model:

Observable	SCE+ED	MPS [4]
E_0	-0.3183098860(2)	-0.318338(24)
M_S/g	1.12837916711(1)	1.1279(12)
M_V/g	0.5641895845(9)	0.56421(9)

- Chiral condensate - massless case:

x	SCE+ED	MPS [5]	Difference
20	-0.189878819389204	-0.19025255847009401	0.00037
25	-0.187519020840406	-0.18796879340592226	0.00045
30	-0.185829589660617	-0.18620821935803569	0.00038
cont.	-0.16(1)	-0.159930(8)	

- C.c. - massive $m = 0.125$ (this is after subtracting log divergence [6]):

x	SCE+ED	MPS [5]
cont.	-0.091(5)	-0.092023(4)

Lattice Hamiltonian approach
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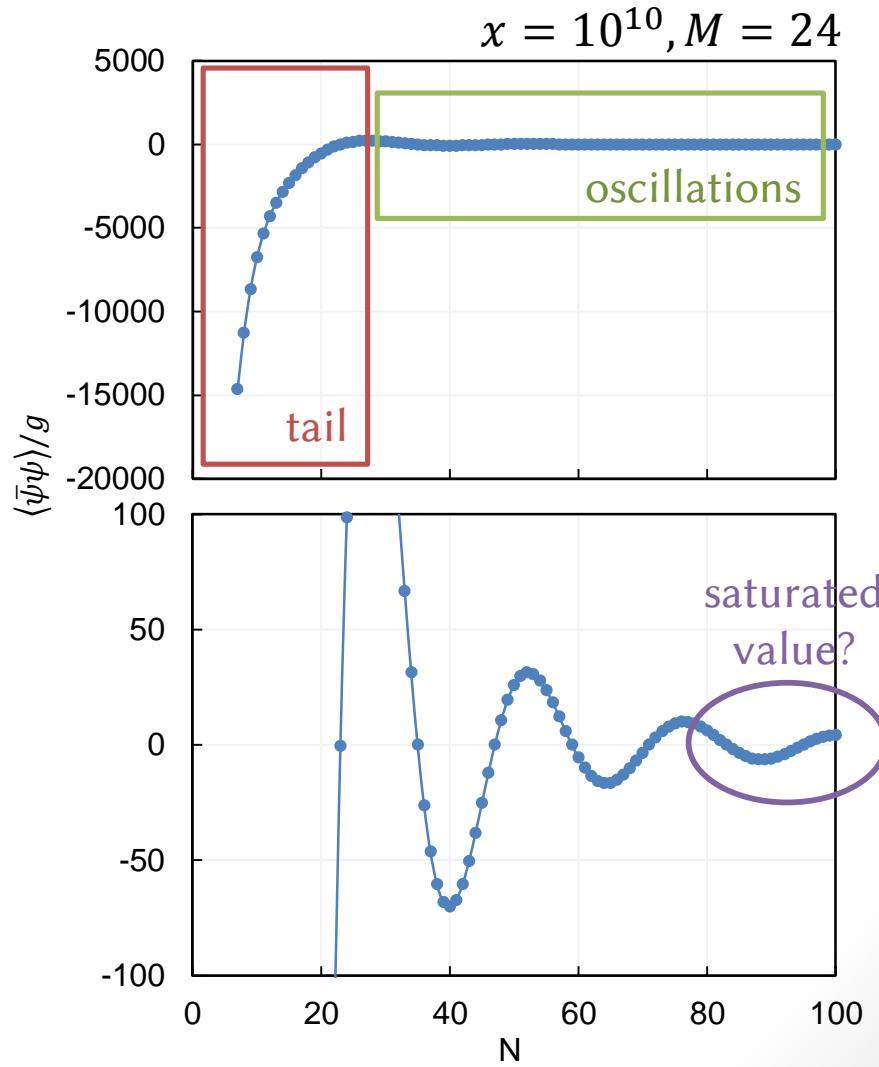
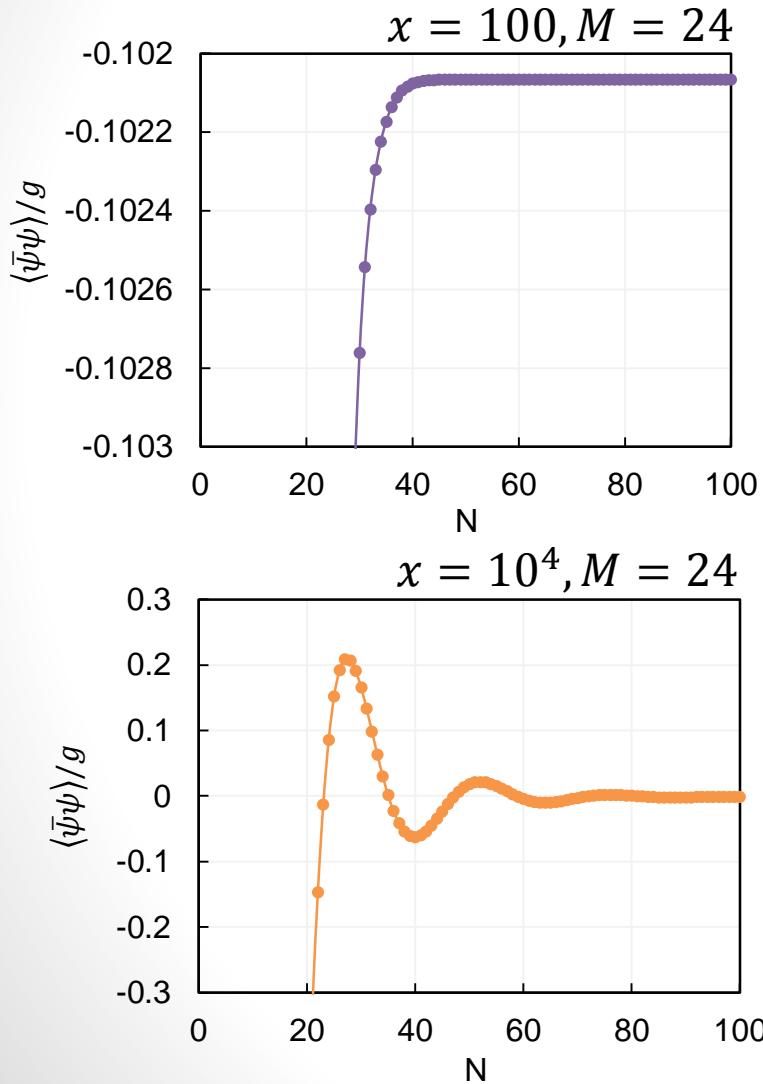
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[4] Bañuls, et al., JHEP 11 (2013) 158..

[5] Bañuls, et al., PoS(Lattice2013)332.

[6] de Forcrand, Nucl.Phys.Proc.Suppl. 63 (1998) 679-681.

Oscillations of chiral condensate while changing the SCE order N



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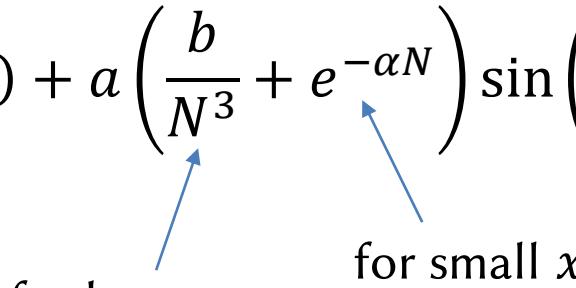


Oscillations: fitting ansatz

- We have chosen the following fitting function:

$$\Sigma(N) = \Sigma(N \rightarrow \infty) + a \left(\frac{b}{N^3} + e^{-\alpha N} \right) \sin \left(\frac{2\pi}{T} N + \varphi \right)$$

for huge x for small x



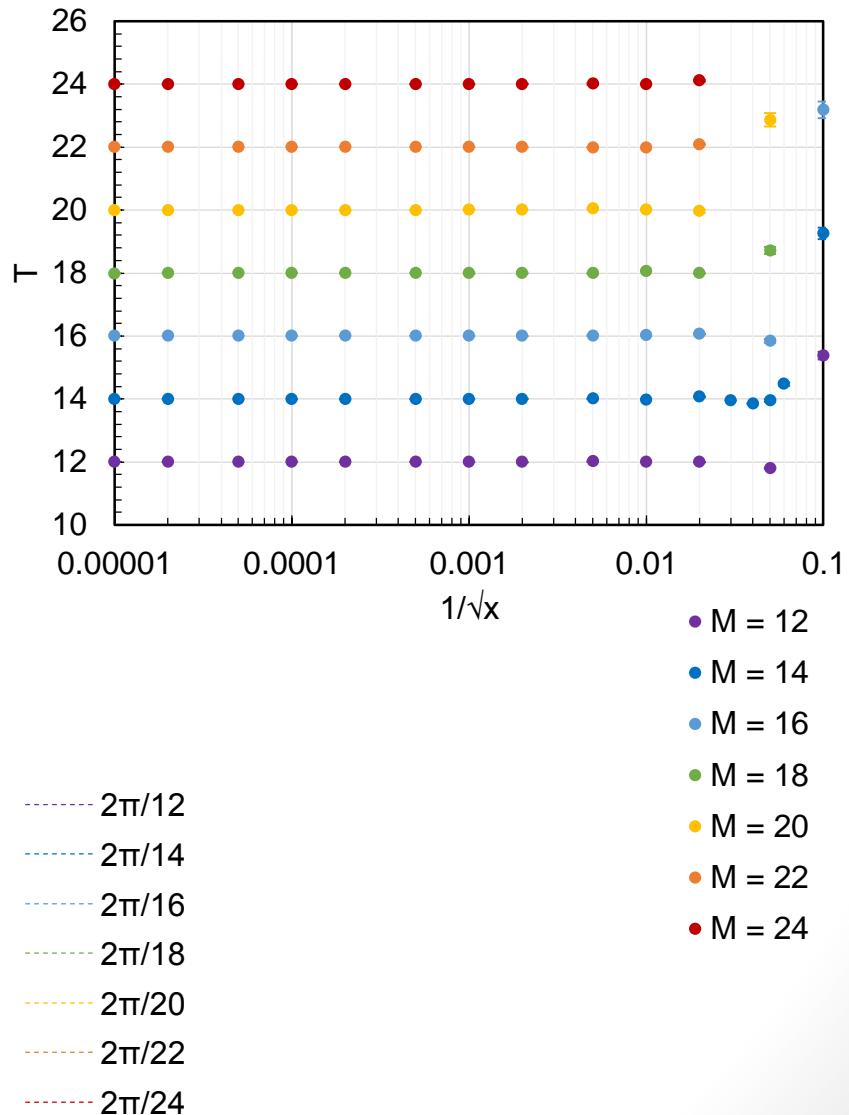
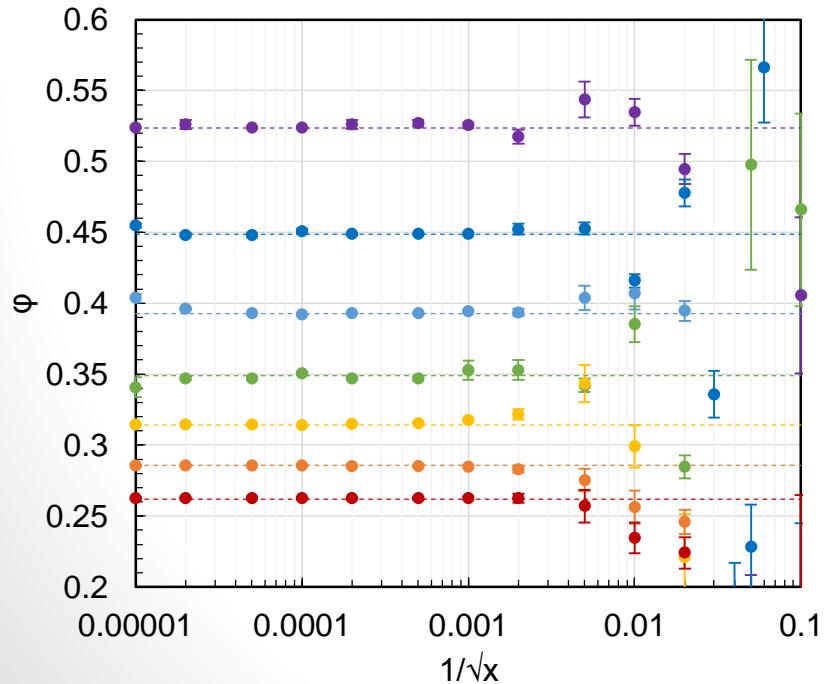
- If we can guess the fitting ansatz correctly, we can use small number of points to approximate saturated values, $\Sigma(N \rightarrow \infty)$.



Oscillations: fitting parameters

Phase and period:

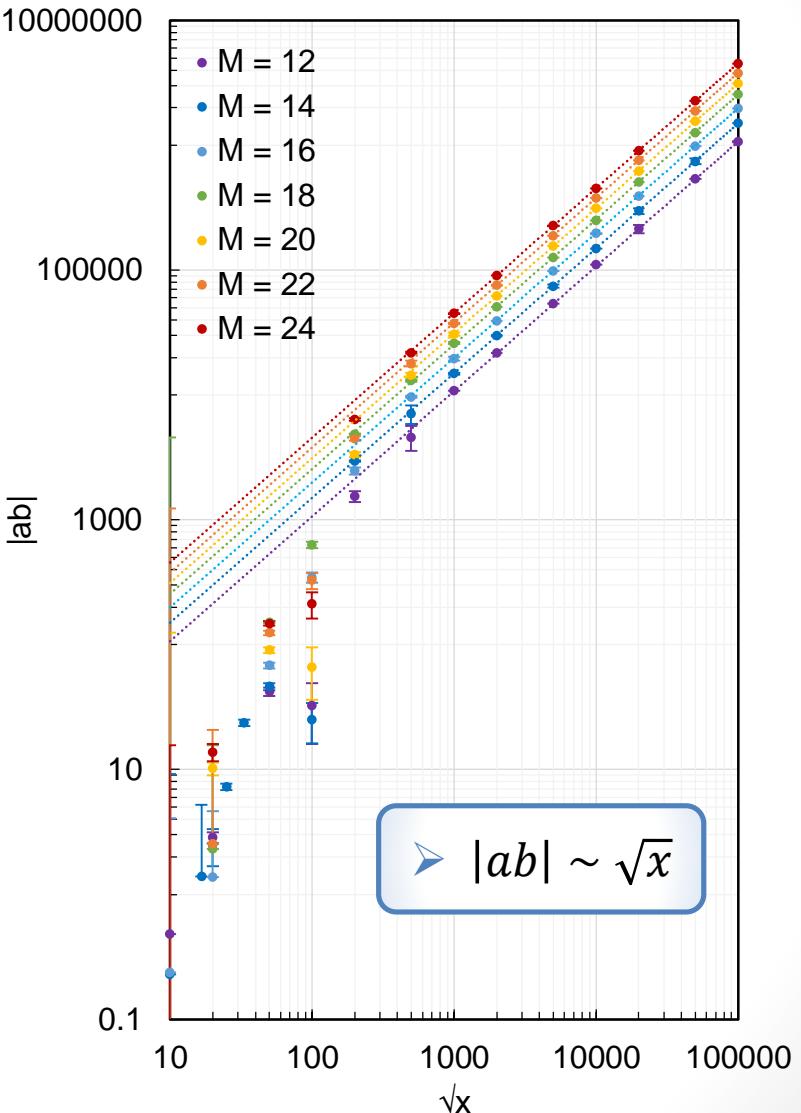
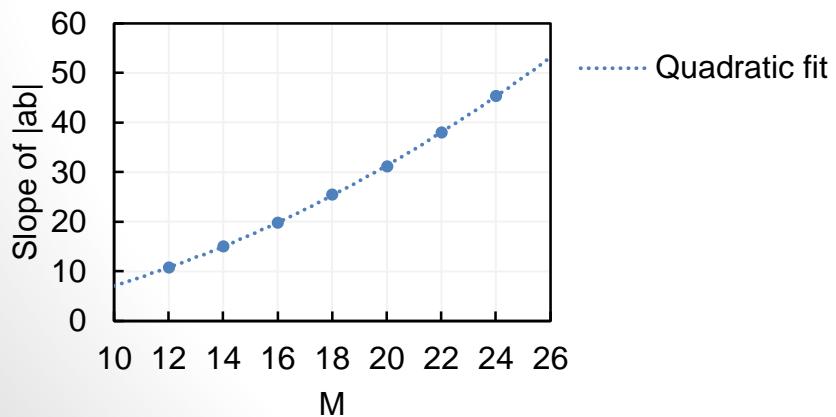
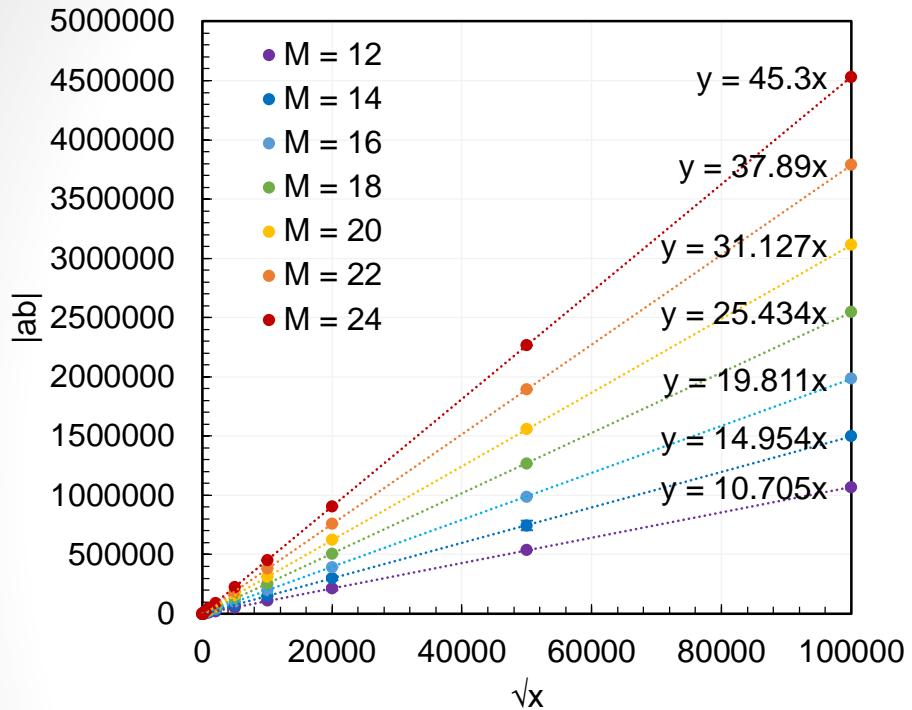
- $T = M$
- $\phi = \frac{2\pi}{M}$



Fitting function: $\Sigma(N) = \Sigma(N \rightarrow \infty) + a \left(\frac{b}{N^3} + e^{-aN} \right) \sin \left(\frac{2\pi}{T} N + \varphi \right)$



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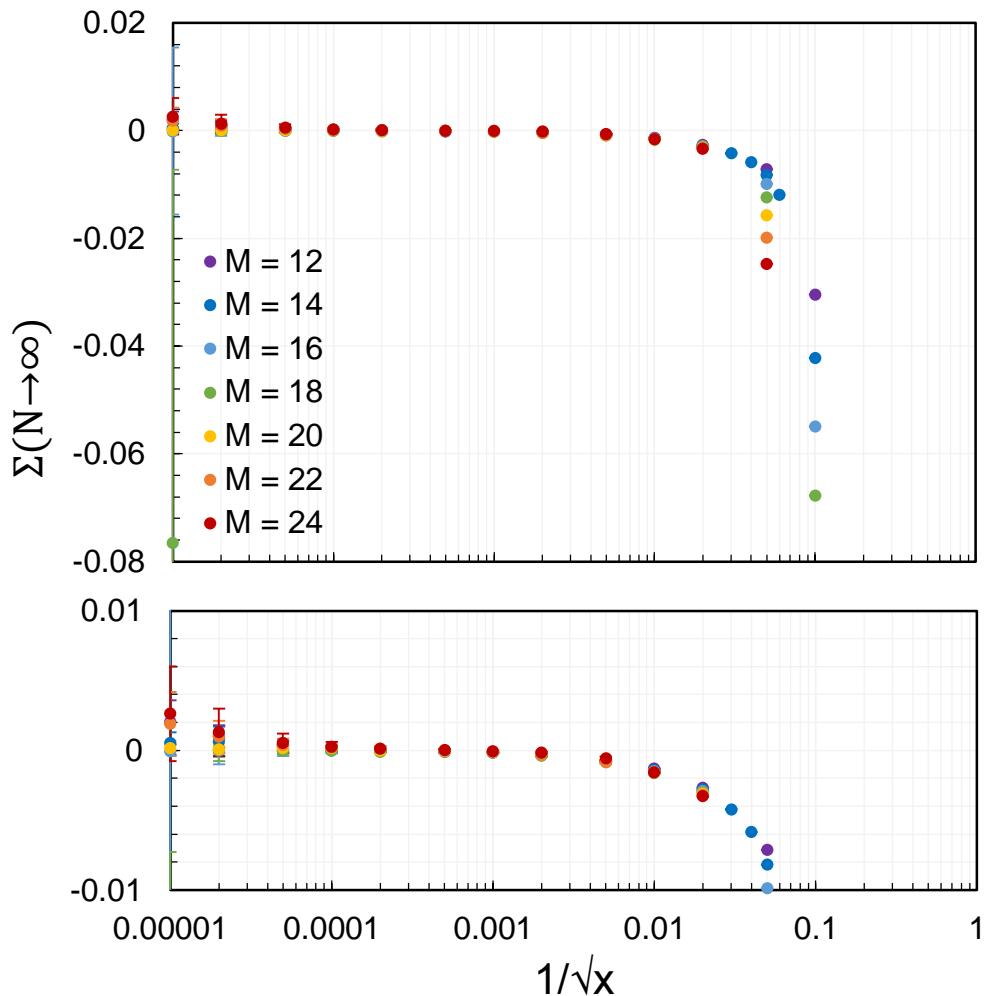


➤ $|ab| \sim \sqrt{x}$



Fitting function: $\Sigma(N) = \Sigma(N \rightarrow \infty) + a \left(\frac{b}{N^3} + e^{-aN} \right) \sin \left(\frac{2\pi}{T} N + \varphi \right)$

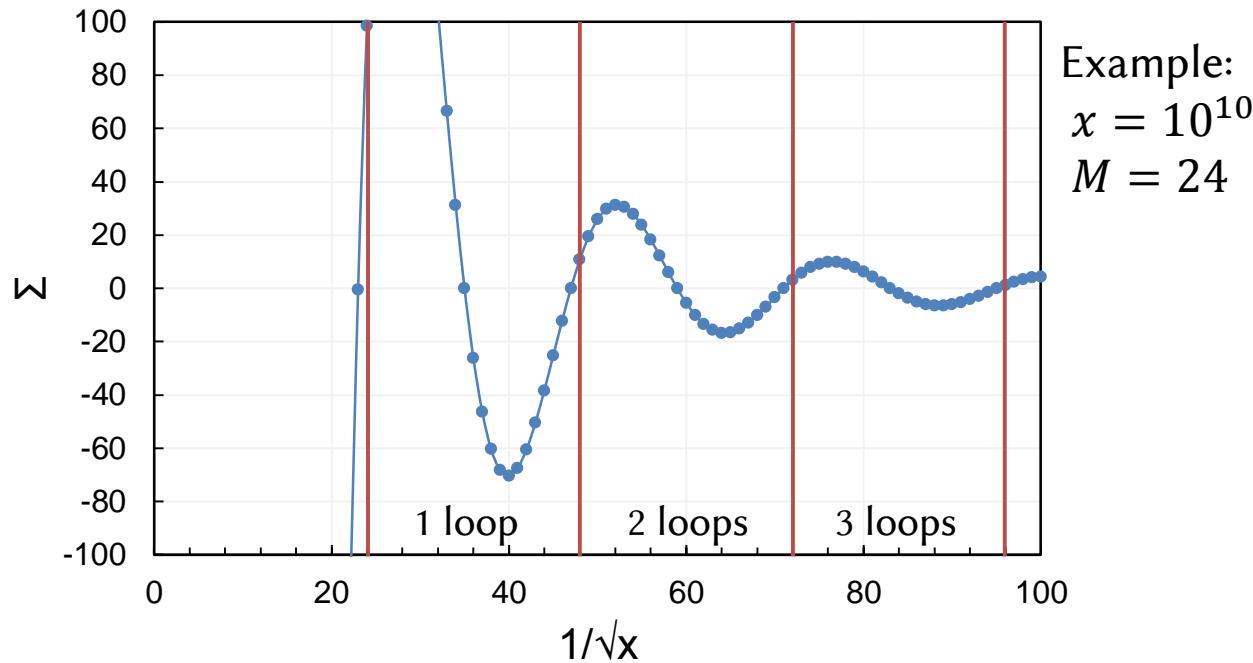
- High x : errors due to huge oscillations
- $\Sigma(N \rightarrow \infty)$ seems to go to zero – because of huge finite volume effects.





Oscillations of the chiral condensate and flux loops

- Every time $N = k M$, we reach the next flux loop in the system
- Period must reflect presence of the flux loops



- Final fitting ansatz:

$$\Sigma(N, M, x) = \Sigma(N \rightarrow \infty, M, x) + \left(A(M) \frac{\sqrt{x}}{N^3} + B(M, x) e^{-\alpha(M, x)N} \right) \sin \frac{2\pi}{M} (N + 1)$$



Summary and outlook

Lattice Hamiltonian results for massless Schwinger model:

- ❖ Almost machine precision for GS energy and mass gaps
- ❖ Chiral condensate, but we have a problem with FVE

Future:

How to deal with
finite volume
effects in the chiral
condensate?

Fitting functions?

Chiral condensate
oscillations –
further
investigation

Fitting function?

Damped harmonic
oscillator for small
 x ?



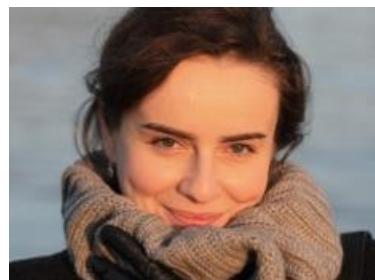
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